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OUTPUT DEADBEAT CONTROL OF
DISCRETE-TIME MULTIVARIABLE SYSTEMS

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by

Jacob C. Engwerda

Abstract

In this note we solve the problem of output deadbeat regulation with internal stability in its full generality. That is, our only assumption is that the system is described by a linear time-invariant recurrence equation. By making an appropriate state-space decomposition we show that the results obtained by Kimura et al. (1981) can be generalized straightforwardly. An advantage of this approach is that it facilitates an easy proof of the basic solvability condition, gives a good insight into the basics of the problem, and provides a synthesis procedure for constructing a minimal-time deadbeat controller. The approach is used to parametrize a set of weight matrices which turn the corresponding minimum variance controller into a minimal-time deadbeat one.

Keywords
Deadbeat control, minimum variance control, state-space decomposition, discrete-time systems.

1. INTRODUCTION

In Marrari et al (1989) a variant of the well known state deadbeat control problem is studied. They discuss the problem to design a state feedback which in minimum time drives the outputs of the system to zero. As they point out this is an interesting theoretical problem which was partly discussed before in Leden (1977), Akashi et al (1978) and Kimura et al (1981).

In Marrari's paper two algorithms are presented which partially solve the problem in case the system is completely reachable. The problem is only partially solved in the sense that their algorithms force the output to zero within a time which is in general smaller than the time a state feedback controller needs to force all states to zero. Their claim that this time period is minimal seems, however, premature. A serious proof is lacking. Since the design of an appropriate weighted minimum variance controller seems to be rather involved it seems reasonable first to answer the question under which conditions a minimal-time deadbeat output regulator, with no restrictions on control structure, exists.

In Kimura et al. (1981) the problem of output deadbeat regulation with internal stability is solved for a linear discrete-time system described by

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_3 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u(k)$$

$$z(k) = D_1 x_1(k) + D_2 x_2(k)$$

under the assumptions that B_1 is monic, D_1 is epic, (D_2,A_2) is observable, (A_1,B_1) is controllable, (D_1,A_1) is observable and the spectrum of A_2, i.e. $\sigma(A_2)$, contains no good modes. It is obvious that by introducing the auxiliary state variable $x^T(k) := (x_1^T(k)\ x_2^T(k))$ we have that this system is a special type of the general discrete-time system $x(k+1) = Ax(k) + Bu(k)$; $y(k) = Cx(k)$.

Though many systems can be described by the model considered by Kimura et al., it is e.g. from a theoretical point of view more satisfactory to have solvability conditions for the output deadbeat regulation problem w.r.t. the general discrete-time system.

Therefore we consider in this paper the output deadbeat regulation with internal stability in its full generality. First we present both necessary and sufficient conditions for the existence of a state feedback controller which drives the output to zero in minimum time, and then give an algorithm for actually constructing such a minimum time deadbeat controller. Apart from the fact that our result is more general than that of Kimura et al., we will see that due to the fact that we consider the state-space representation in its full generality we can now optimally exploit a state-space decomposition technique in actually constructing a set of minimal-time deadbeat controllers. Subsequently we show that this set of controllers can also be obtained by making appropriate choices for the weight matrix in the minimum variance controller.

2. PRELIMINARIES

In this section we formulate the problem and introduce some notation and well-known concepts.
The problem reads as follows.

<u>Deadbeat-output State-feedback Problem with Internal stability</u>

Given the finite-dimensional, linear, time-invariant discrete-time system described by the difference equation

$$x(k+1) = A\,x(k) + B\,u(k); \quad x(0) = x$$
$$y(k) = C\,x(k), \tag{1}$$

where $x(.) \in \mathbb{R}^n$, $u(.) \in \mathbb{R}^m$, $y(.) \in \mathbb{R}^q$.
Find a linear state feedback map $u(k) = F\,x(k)$ such that the outputs are forced to zero and kept zero for any initial state x in a minimum number of time steps μ, i.e.
$y(k) = C\,x(k) = C(A+BF)^k\,x = 0\ \forall x$ and $k \geq \mu$, with μ as small as possible. Moreover, we require that $\sigma(A+BF) \subset \mathbb{C}_g$, where $\mathbb{C}_g$ is some prespecified part of the complex plane containing zero. The smallest possible integer for which this can be accomplished is referred to as the minimal settling

time, and the corresponding map a minimal-time controller. Note that this integer may depend on the choice of $\mathbb{C}_g$.

Note that whenever a controller exists which makes the output deadbeat, a minimal-time controller exists.

To solve this problem we make a state space decomposition of system (1). This decomposition uses the notions of the reachability and a particular stabilizability subspace. We formalize these concepts in definition 1. That they are well defined can be found e.g. in Hautus et al. (1980).

Definition 1

- The reachability subspace denoted by $\langle A|\mathrm{Im}\, B\rangle$ equals $\mathrm{Im}\, B + A\, \mathrm{Im}\, B + \ldots + A^{n-1}\, \mathrm{Im}\, B$, where Im B is the image of matrix B.
- V is called a stabilizability subspace (w.r.t. $\mathbb{C}_g$) if for any $x(0) \in V$ exists a Bohl function u(.) such that $x(0,k,x(0),u) \in V$ for all $k \geq 0$ and $x(0,.,x(0),u)$ is stable (w.r.t. $\mathbb{C}_g$).
- $V_g^*(\mathrm{Ker}\, C)$ denotes the largest stabilizability subspace (w.r.t. $\mathbb{C}_g$) that is contained in the Kernel (Ker) of C.
- A subspace V is called M-invariant if $MV \subset V$.
- A subspace V is called controlled invariant if $AV \subset V + \mathrm{Im}\, B$.
- An eigenvalue λ is called controllable if rank $[A-\lambda I|B] = n$.
- An eigenvalue λ is called observable if rank $\begin{bmatrix} A-\lambda I \\ C \end{bmatrix} = n$. □

The next two lemmas w.r.t. the reachability subspace and the largest stabilizability subspace are well known (see e.g. Wonham (1979)).

Lemma 2

i) $\langle A|\mathrm{Im}\, B\rangle$ is A-invariant.

ii) $V_g^*(\mathrm{Ker}\, C)$ is controlled invariant. □

Lemma 3

A subspace V is a stabilizability subspace w.r.t. $\mathbb{C}_g$ if and only if (iff) there exists a map F satisfying

i) $(A+BF)\, V \subset V$ and ii) $\sigma(A+BF|V) \subset \mathbb{C}_g$. □

For the derivation of results it will appear to be convenient to apply first a feedforward $u = Kx + u'$ to system (1), where K makes $V_g^*(\mathrm{Ker}\ C)$ A+BK-invariant and the spectrum of $A + BK$ w.r.t $V_g^*(\mathrm{Ker}\ C)$ is contained in $\mathbb{C}_g$. So, without loss of generality we can assume that the spectrum of $A + BK$ w.r.t. $V_g^*(\mathrm{Ker}\ C) \cap \langle A|\mathrm{Im}\ B\rangle$ is contained in $\mathbb{C}_g$.
So instead of system (1) we consider

$$
\begin{aligned}
x'(k+1) &= (A+BK)\ x'(k) + B\ u'(k); \quad x'(0) = x \\
y(k) &= C\ x'(k)
\end{aligned}
\tag{2}
$$

Now, consider the following state space decomposition.

$$X_1 := \langle A|\mathrm{Im}\ B\rangle \cap V_g^*(\mathrm{Ker}\ C)$$

$$X_1 \oplus X_2 := V_g^*(\mathrm{Ker}\ C)$$

$$X_1 \oplus X_3 := \langle A|\mathrm{Im}\ B\rangle$$

$$X_1 \oplus X_2 \oplus X_3 \oplus X_4 := \mathbb{R}^n.$$

Application of the invariance properties from lemmas 2 and 3 yields then the next state space decomposition.

Theorem 4
Let $\mathbb{R}^n = X_1 \oplus X_2 \oplus X_3 \oplus X_4$.
With a basis adapted to this decomposition system (2) is described by the next recurrence equation

$$
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & A_{22} & 0 & A_{24} \\ 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & A_{44} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \\ B_3 \\ 0 \end{bmatrix} u(k)
$$

$$y(k) = (0\ 0\ C_3\ C_4) \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}$$

where $\left[\begin{bmatrix} A_{11} & A_{13} \\ 0 & A_{33} \end{bmatrix}, \begin{bmatrix} B_1 \\ B_3 \end{bmatrix}\right]$ is reachable, and $\sigma(A_{11}) \subset \mathbb{C}_g$ □

Using this basic result we show in the next section how the general problem can be solved in a straightforward way.

3. PROBLEM SOLUTION

In this section we solve the central problem of this paper.
We start with giving a necessary and sufficient condition for the existence of a minimal-time controller. This condition generalizes the result obtained by Kimura et al. (theorem 1, 1981), where the same problem was considered for the system as mentioned in the introduction.

Theorem 5
There exists a minimal-time deadbeat-output state-feedback controller iff $\sigma[A \mid \mathbb{R}^n/(\langle A|\mathrm{Im}\ B\rangle + V_g^*(\mathrm{Ker}\ C))] = \{0\}$.

Proof
"<u>only if</u>": Consider the state-space decomposition introduced in theorem 4. From this decomposition it is clear that
$\sigma[A \mid \mathbb{R}^n/(\langle A|\mathrm{Im}\ B\rangle + V_g^*(\mathrm{Ker}\ C))] = \sigma(A_{44})$ (note that this is independent of the chosen preliminary feedback in system (2)!).
Now, let λ be an eigenvalue of A_{44}. Then λ is also an eigenvalue of A. Since the eigenvalue λ can not be removed by feedback, it must belong to $\mathbb{C}_g$.
Now, let x be an eigenvector corresponding with $\lambda \neq 0$. Then $CA^k x = \lambda^k Cx = 0$. Consequently $x \in V_g^*(\mathrm{Ker}\ C)$, which contradicts the assumption that $\lambda \in \sigma(A_{44})$. So the stated condition is a necessary one.

"<u>if</u>": To prove that the condition is sufficient too, we take $F = (0\ 0\ F_3\ 0)$, where F_3 is such that $A_{33} + B_3F_3$ is nilpotent (this is possible since (A_{33},B_3) is reachable). Using this state feedback controller simple calculation shows that the system becomes deadbeat-output, which completes the proof. □

We stress here that the theorem merely states an existence result. In general the resulting time index from whereon the output is always zero will not be minimal for the chosen state-feedback controller.
To obtain a characterization for the minimal settling time we proceed with an algorithm based on the analysis performed by Kimura et al. (1981).
Consider the following sequence of subspaces $T_0, T_1, \ldots$

$$T_0 := V_g^*(\mathrm{Ker}\ C)$$

$$T_i := A^{-1}(T_{i-1} + \mathrm{Im}\ B),\ i \geq 1. \tag{3}$$

The interpretation of subspace T_i is that it contains all initial states which can be brought in T_0 at time i, by applying some input sequence $u(0), u(1), \ldots, u(i-1)$.
We have the following property from which easily an estimate is obtained for the minimal settling time.

Lemma 6
Assume that there exists a minimal-time deadbeat-output state-feedback controller K and the minimal settling time is k.
Then $T_k = \mathbb{R}^n$.

Proof
We show by induction on i that $\mathrm{Im}\ (A+BK)^{k-i} \subset T_i$, $i = 0,1,\ldots,k$. (i)
If $i = 0$, we have to prove that $\mathrm{Im}\ (A+BK)^k \subset V_g^*(\mathrm{Ker}\ C)$. This inclusion, however, is trivially satisfied since by assumption $C(A+BK)^k = 0$ and $\sigma(A+BK) \subset \mathbb{C}_g$.
So, assume now that the inclusion holds for $i = \tau$, i.e. $\mathrm{Im}\ (A+BK)^{k-\tau} \subset T_\tau$.
From this inclusion we immediately deduce that $(A+BK)\ \mathrm{Im}\ (A+BK)^{k-\tau-1} \subset T_\tau$, which implies that we also have $\mathrm{Im}\ (A+BK)^{k-\tau-1} \subset A^{-1}(T_\tau + \mathrm{Im}\ B)$.

Since by definition $A^{-1}(T_\tau + \text{Im } B)$ equals $T_{\tau+1}$, the induction argument is completed with this.
The stated result follows now by taking $i = k$ in (i). □

Corollary 7
The minimal settling time is not smaller than $k_s := \min_i \{i \mid T_i = \mathbb{R}^n\}$. □

In fact we can show that the minimal settling time always equals this number k_s. This is the contents of the next algorithm.

Algorithm 8
Let the subspace $\tilde{T}_i$, $i=0,\dots,k$ be defined als follows;

$$\tilde{T}_0 := T_0$$

$$T_{i-1} \oplus \tilde{T}_i := T_i, \; i=1,\dots,k_s.$$

Next define maps G_i satisfying

$$G_0 := 0, \text{ and}$$

$$(A+BG_i)\, \tilde{T}_i \subset T_{i-1} \; i=1,\dots,k_s. \tag{i}$$

Then any map G defined by

$$G|\tilde{T}_i = G_i|\tilde{T}_i \; i=0,\dots,k_s$$

solves the minimal time deadbeat-output state-feedback control problem.

Proof
The proof is according the lines of theorem 2 in Kimura et al. (1981). First we note that since $V_g^*(\text{Ker } C)$ is controlled invariant it is easily shown that the inclusion $T_{i-1} \subset T_i$ $i=1,\dots,k_s$ holds. So, the definition of $\tilde{T}_i$ makes sense. Moreover, since $A\,\tilde{T}_i \subset T_{i-1} + \text{Im } B$ it is clear that always a map G_i exists satisfying (i).
So what is left to be shown is that $(A+BG)^{k_s} \subset V_g^*(\text{Ker } C)$.

To that end we note first that

$$(A+BG)\ T_i = (A+BG)(\tilde{T}_0+\ldots+\tilde{T}_i) \subset \sum_{j=1}^{i} (A+BG_j)\ \tilde{T}_j \subset \sum_{j=1}^{i-1} T_j \subset T_{i-1}.$$

Consequently $(A+BG)^{k_s}\ T_{k_s} \subset (A+BG)^{k_s-1}\ T_{k_s-1} \subset \ldots \subset V_g^*(\mathrm{Ker}\ C)$.

As $T_{k_s} = \mathbb{R}^n$, this completes the proof. □

Combination from results of lemma 6 and algorithm 8 then yields the next theorem.

Theorem 9

The minimal settling time equals $\min_i \{i \mid T_i = \mathbb{R}^n\}$. □

4. THE RELATIONSHIP WITH LEAST SQUARES MINIMIZATION

In this section we consider the connection between Minimum Variance (MV) control and the deadbeat-output state-feedback control problem.

Using the results of the previous section we show that there always exists a minimal-time deadbeat-output state-feedback controller within the set of controllers resulting from the minimization of the MV cost criterion $J := x^T(k)Qx(k)$ w.r.t. $\Sigma : x(k+1) = Ax(k+1) + Bu(k)$, by making an appropriate choice of the weight matrix Q.

In fact we will first parametrize a set of deadbeat-output state-feedback controllers which satisfy the requirement that every state component is controlled as fast as possible to zero (in the sense as discussed in note 15).

Then we use this parametrization to present a set of minimal-time deadbeat-output MV-controllers.

The following lemma is crucial in deriving our results.

Since its proof is somewhat technical we defer it to the appendix. In the lemma the notion of Jordan block is used. We recall from literature that a matrix $D \in \mathbb{R}^{n\times n}$ is called a Jordan block of order n if it has the following form: $\begin{bmatrix} a & 1 & & \\ & \ddots & \ddots & \\ & & a & 1 \\ & & & a \end{bmatrix}$.

Lemma 10

Let $A := \begin{bmatrix} J_1 & D \\ 0_{m\times n} & J_2 \end{bmatrix}$, where J_1, J_2 are nilpotent Jordan blocks of order n and m, respectively, $0_{m\times n} \in \mathbb{R}^{m\times n}$ is the zero matrix and $D \in \mathbb{R}^{n\times m}$ has the following form:

$$D := \begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \vdots & & \\ d_{n-11} & \cdots & d_{n-1m} \\ f_1 & \cdots & f_m \end{bmatrix}$$

Denote $\max(n,m)$ by $\bar{r}$, $\min(n,m)$ by $\underline{r}$, $\max(0,m-n)$ by $\bar{s}$ and $\max(0,n-m)$ by $\underline{s}$. Then

$$A^{\bar{r}} = \left[\begin{array}{c|c|c} 0_{n\times n} & 0_{n\times\bar{s}} & \bar{D}_{\bar{r}} \\ \cline{2-3} & & 0_{\underline{s}\times m} \\ \hline 0_{m\times n} & \multicolumn{2}{c}{0_{m\times m}} \end{array}\right]$$

where $\bar{D}_{\bar{r}} \in \mathbb{R}^{(m-\bar{s})\times(n-\underline{s})}$ equals

$$\begin{bmatrix} f_1 & f_2+d_{n-11} & f_3+d_{n-12}+d_{n-21} & \cdots & f_{\underline{r}} + \sum_{i=1}^{\underline{r}-1} d_{n-i\underline{r}-i} \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & f_3+d_{n-12}+d_{n-21} \\ & & & \ddots & f_2+d_{n-11} \\ 0 & \cdot & \cdot & 0 & f_1 \end{bmatrix}$$

(Note that either $0_{n\times\underline{s}}$ or $0_{\underline{s}\times m}$ does not exist and $D_{\bar{r}}$ is always a square matrix.) □

Corollary 11

Assume that in lemma 10 the parameters $f_1,\dots,f_m$ can be arbitrarily chosen.

Then A is nilpotent of minimal order $\bar{r}$ iff $f_1 = 0$, $f_2 = -d_{n-11},\dots,$ $f_{\underline{r}} = -\sum_{i=1}^{\underline{r}-1} d_{n-i\underline{r}-i}$.

Proof

That the order of nilpotency of A is not less than $\bar{r}$ is obvious.

On the other hand it is clear from lemma 10 that we get $A^{\bar{r}} = 0$ iff the parameters $f_1,\dots,f_{\underline{r}}$ are chosen as stated above. □

Remark 12

It is easily verified that if the Jordan block J_2 in lemma 10 is replaced by a nilpotent Jordan matrix containing k Jordan blocks J_i of order p_i, $i = 1,\dots,k$, the content of this lemma remains almost the same. All what changes is that the right-upper block of matrix $A^{\bar{r}}$ is replaced by k $n\times p_i$-blocks of that form. □

The above promised parametrization of all deadbeat-output controllers is obtained by transforming the transformed system discussed in theorem 4 once again. To avoid some technicalities we assume, without loss of generality, that matrix B_3 in theorem 3 is injective. We have:

Lemma 13

There exists a similarity transformation S such that system (2) is described by the next recurrence equation:

$$\begin{bmatrix} \bar{x}_1(k+1) \\ \bar{x}_2(k+1) \\ \bar{x}_3(k+1) \\ \bar{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \bar{A}_{13} & \bar{A}_{14} \\ 0 & A_{22} & 0 & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{33} & \bar{A}_{34} \\ 0 & 0 & 0 & \bar{A}_{44} \end{bmatrix} \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \\ \bar{x}_3(k) \\ \bar{x}_4(k) \end{bmatrix} + \begin{bmatrix} \bar{B}_1 \\ 0 \\ \bar{B}_3 \\ 0 \end{bmatrix} \bar{u}(k) \tag{4}$$

$$y(k) = (0 \quad 0 \quad \bar{C}_3 \quad \bar{C}_4)\begin{bmatrix}\bar{x}_1(k)\\ \bar{x}_2(k)\\ \bar{x}_3(k)\\ \bar{x}_4(k)\end{bmatrix},$$

where $\left[\begin{bmatrix}A_{11} & \bar{A}_{13}\\ 0 & \bar{A}_{33}\end{bmatrix}, \begin{bmatrix}\bar{B}_1\\ \bar{B}_2\end{bmatrix}\right]$ is reachable, $\sigma(A_{11}) \subset \mathbb{C}_g$, and $\bar{A}_{33}$, $\bar{B}_3$ and $\bar{A}_{44}$ are as follows:

$$\bar{A}_{33} := (E_{i1}) \; ; \quad \bar{B}_3 := (\underline{e}_{1i}), \text{ with}$$

$$E_{i1} := \begin{array}{c} \\ n_i \left\{ \vphantom{\begin{bmatrix}0\\ \vdots\\ 0\\ a\end{bmatrix}} \right. \end{array} \begin{array}{c} \overbrace{\hphantom{0 \; \cdots \; 0}}^{\underline{n}_i} \\ \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & & 0\\ \vdots & & & & \ddots & \ddots & & \vdots\\ 0 & \cdots & & & 0 & 1 & 0 \cdots & 0\\ a_{i1} & \cdot & \cdot & & & \cdot & & a_{in} \end{bmatrix} \end{array}$$

and $\underline{e}_{1i}$ the $(n_1+\ldots+n_i)^{th}$ standard unit vector in $\mathbb{R}^n$, $i = 1,\ldots,\ell$ (here $\underline{n}_i := n_0 + \ldots + n_i+1$, with $n_0 := 0$); $\bar{A}_{44} := \begin{bmatrix}J_1 & & \\ & \ddots & \\ & & J_k\end{bmatrix}$, with $J_i \in \mathbb{R}^{p_i \times p_i}$ nilpotent Jordanblocks or order p_i.

Here $n_1 \geq n_2 \geq \ldots \geq n_\ell \geq 1$ with $\sum_{i=1}^{\ell} n_i = n$ are the controllability indices, with n_1 as "the" controllability index, a_{ij} denotes an arbitrary real number, and $p_1 \geq \ldots \geq p_k \geq 1$ with $\sum_{i=1}^{k} p_i = m$.

Proof

Since (A_{33},B_3) is reachable we have from Luenberger (1967) that there exist non-singular transformation matrices S_1 and T such that $\bar{A}_{33} = S_1A_{33}S_1^{-1}$ and $\bar{B}_3 = S_1B_3T$.

Furthermore, since A_{44} is nilpotent, we know that there exists a non-singular transformation matrix S_2 such that $\bar{A}_{44} = S_2A_{44}S_2^{-1}$.

Applying the similarity transformation $x(k) := \begin{bmatrix} I & & & \\ & I & & \\ & & S_1^{-1} & \\ & & & S_2^{-1} \end{bmatrix} \bar{x}(k)$ and input transformation $u(k) = T\bar{u}(k)$ yields then the advertised result. □

Theorem 14

Consider the transformed system (4). Let $\bar{A}_{34} := \begin{bmatrix} d_{11} & \cdots & d_{1m} \\ \vdots & & \vdots \\ d_{n1} & \cdots & d_{nm} \end{bmatrix}$.

For this system $\bar{u}(k) = (0\ 0\ F_3\ F_4)\bar{x}(k)$ is a minimal-time deadbeat-output controller if

$$F_3 := \begin{bmatrix} -a_{11} & \cdots & -a_{1n} \\ \vdots & & \vdots \\ -a_{\ell 1} & \cdots & -a_{\ell n} \end{bmatrix} \text{ and } F_4 := (f_{st}),\ s = 1,\ldots,\ell,\ t = 1,\ldots,k,$$

where $f_{st} \in \mathbb{R}^{1\times p_s}$ is given by

$$f_{st} := (-d_{\underline{n}_s \underline{p}_{t-1}+1} \cdots - \sum_{i=1}^{\underline{r}_{st}-1} d_{\underline{n}_s - i\,\underline{p}_{t-1}+\underline{r}_{st}-i} \quad * \cdots *)$$

Here $\underline{n}_s := \sum_{i=1}^{s} n_i$; $\underline{p}_i := \sum_{j=1}^{i} p_j$; $\underline{p}_0 := 0$; $\underline{r}_{ij} := \min(n_i, p_j)$ and '*' denotes a parameter that can be chosen arbitrarily.
The minimal settling time is equal to $\max(n_1, p_1)$.

Proof

From (3) it is clear that there exist states in X_3 which can be brought in $V_g^*(\ker C)$ by applying an appropriate input sequence whose minimal length equals the controllability index n_1 of $(\bar{A}_{33}, \bar{B}_3)$. Similarly it is obvious too that there exist states in X_4 which, whatever control sequence we use, have a minimal settling time of p_1. In other words, the minimal settling time is at least $\max(n_1, p_1)$.
Using the results of corollary 11 (and remark 12) it is now a matter of straightforward calculation to show that the advertised feedback controller F yields a closed-loop matrix of the form

$$\begin{bmatrix} A_{11} & A_{12} & \bar{A}_{13}+\bar{B}_1F_3 & \bar{A}_{14}+\bar{B}_1F_4 \\ 0 & A_{22} & 0 & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{33}+\bar{B}_3F_3 & \bar{A}_{34}+\bar{B}_3F_4 \\ 0 & 0 & 0 & \bar{A}_{44} \end{bmatrix}, \qquad (5)$$

where the matrix $\begin{bmatrix} \bar{A}_{33}+\bar{B}_3F_3 & \bar{A}_{34}+\bar{B}_3F_4 \\ 0 & \bar{A}_{44} \end{bmatrix}$ is nilpotent of order $\max(n_1,p_1)$.
So, this controller makes the system deadbeat in $\max(n_1,p_1)$ time-steps and moreover does not affect the eigenvalues of A_{11} and A_{22}. Consequently it is an appropriate minimal-time deadbeat controller. □

Note 15

Note that the construction of corollary 11 shows that if we consider the following decomposition of the state variables $\begin{bmatrix} \bar{x}_3 \\ \bar{x}_4 \end{bmatrix}^T =: (v_1^T v_2^T \ldots v_\ell^T w_1^T \ldots w_k^T)$, where each vector v_i, $i = 1,\ldots,\ell$, contains n_i variables and w_i, $i = 1,\ldots,k$, contains p_i variables, then all variables v_i and w_i are minimal deadbeat (i.e. $v_i(t) = 0$ $\forall\bar{x}$ and $t \geq s_i$, with s_i as small as possible, $i = 1,\ldots,\ell$ (and similarly for w_i)). □

We will now link the obtained results with MV deadbeat output control by suggesting an appropriate choice of the weight matrix for the MV controller. To that end we first prove a preliminary result.

Lemma 16

Consider system (1) (with, without loss of generality, B injective) and a system obtained from (1) by applying a state transformation $\bar{x} = Sx$ and input transformation $\bar{u} = Tu$. Assume that system (1) and its transformation both are controlled by a MV-regulator with weight matrix Q and $\bar{Q}$, respectively.
Then both closed-loop systems are identical if $\bar{Q} = SQS^{-1}$.

It is easily verified that the closed-loop system of the MV-controlled system (1) equals $x(k+1) = (I-B(B^TQB)^{-1}B^TQ)Ax(k)$. (i)

For the transformed closed-loop system we similarly get $\bar{x}(k+1) = (I-\bar{B}(\bar{B}^T\bar{Q}\bar{B})^{-1}\bar{B}^T\bar{Q})\bar{A}\bar{x}(k)$. Since $\bar{B} = SBT$, $\bar{A} = SAS^{-1}$, $\bar{x} = Sx$ and $\bar{Q} = SQS^{-1}$ it is easily seen by substitution of these transformed variables into the transformed closed-loop expression that we reobtain the closed-loop system (i). □

A consequence of this lemma is that to construct an appropriate minimal-time output-deadbeat MV-controller it suffices to design such a controller for our transformed system (4).

Theorem 17

With the notation of lemma 13 and theorem 14 we have:
if the weight matrix in the MV-controller is chosen as follows:

$$Q := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & Q_{34} \\ 0 & 0 & Q_{34}^T & Q_{44} \end{bmatrix},$$

where Q_{44} can be chosen arbitrarly and Q_{34} has the following structure: $Q_{34} := (\tilde{Q}_{st})$. $s = 1,\dots,\ell$, $t = 1,\dots,k$, with

$$\tilde{Q}_{st} := n_s\left\{\overbrace{\begin{bmatrix} * & \cdots & \cdot & & & * \\ \vdots & & & & & \vdots \\ * & \cdots & \cdot & & \cdots & * \\ d_{\underline{n}_s-1\,\underline{p}_{t-1}+1} & \cdots & \sum_{i=1}^{\underline{r}_{st}-1} d_{\underline{n}_s-i\,\underline{p}_{t-1}+\underline{r}_{st}-i} & * & \cdots & * \end{bmatrix}}^{p_t}\right.$$

where we used $\underline{n}_s$ to denote $\sum_{i=1}^{s} n_i$, and $\underline{p}_i$ to denote $\sum_{j=1}^{i} p_j$, $i = 1,\dots,t-1$ and $\underline{p}_0 := 0$.

Then the MV-controller yields a closed-loop system that is identical to (5). Consequently this MV-controller is minimal-time output-deadbeat too and has the same additional transient deadbeat minimality property as mentioned in remark 15. □

Before we proceed with the proof of this theorem we first note that matrix Q_{34} is obtained from F_4 in theorem 14 as follows:

1) Consider $-F_4$. Drop the first term of every element in this matrix and consecutively, shift each p_i-block one to the left and add at the end of each block an arbitrary element (so that it contains again p_i-elements).
2) Choose row number n_i of Q_{34} equal to row number i of the matrix constructed in 1), and all other rows of Q_{34} arbitrarily.

Proof (theorem 17)

It is easily verified that using MV-control with Q as indicated above yields the following closed-loop system matrix

$$\begin{bmatrix} A_{11} & A_{12} & \bar{A}_{13}-\bar{B}_1\bar{B}_3^T\bar{A}_{33} & \bar{A}_{14}-\bar{B}_1\bar{B}_3^T\bar{A}_{34}-\bar{B}_1\bar{B}_3^TQ_{34}\bar{A}_{44} \\ 0 & A_{22} & 0 & \bar{A}_{24} \\ 0 & 0 & (I-\bar{B}_3\bar{B}_3^T)\bar{A}_{33} & (I-\bar{B}_3\bar{B}_3^T)\bar{A}_{34}-\bar{B}_3\bar{B}_3^TQ_{34}\bar{A}_{44} \\ 0 & 0 & 0 & \bar{A}_{44} \end{bmatrix}$$

Note that the structure of this matrix coincides with that of (5).
Taking a more closer look at both matrices gives that if we can show that $F_3 = -\bar{B}_3^T\bar{A}_{33}$ and $F_4 = -\bar{B}_3^T\bar{A}_{34} - \bar{B}_3^TQ_{34}\bar{A}_{44}$, then both matrices are in fact identical.
We first show now the equality $F_3 = -\bar{B}_3^T\bar{A}_{33}$. To that end consider an arbitrary row j of F_3. By definition this row equals $(-a_{j1}\ldots -a_{jn})$. Next consider row number j of $\bar{B}_3^T\bar{A}_{33}$. This row equals row number j of $\bar{B}_3^T$ multiplied by $\bar{A}_{33}$. It is easily verified that this product yields row number $n_1+\ldots+n_j$ of $\bar{A}_{33}$, i.e. $(a_{j1}\ldots a_{jn})$, which completes this part of the proof.
In a similar way we prove the second equality $F_4 = -\bar{B}_3^T\bar{A}_{34}-\bar{B}^T{}_3Q_{34}\bar{A}_{44}$.
Note that row number j of $\bar{B}_3^T\bar{A}_{34}+\bar{B}^T{}_3Q_{34}\bar{A}_{44}$ (see argument above) equals the sum of row number $n_1+\ldots+n_j$ of the matrices $\bar{A}_{34}$ and $Q_{34}\bar{A}_{44}$, i.e.
$(d_{j1}\ldots d_{jm})$ +

$$\Bigg[\underbrace{0 \; d_{\underline{j}-1\,1} \cdots \sum_{i=1}^{\underline{r}_{j1}-1} d_{\underline{j}-i\,\underline{r}_{j1}-i} \; * \cdots *}_{p_1} \; \underbrace{0 \; d_{\underline{j}-1\,p_1+1} \cdots \sum_{i=1}^{\underline{r}_{j2}-1} d_{\underline{j}-i\,p_1+\bar{r}_{j2}-i} \; * \cdots *}_{p_2}$$

$$\underbrace{0 \; d_{\underline{j}-1\,\underline{p}_{k-1}+1} \cdots \sum_{i=1}^{\underline{r}_{jk}-1} d_{\underline{j}-i\,\underline{p}_{k-1}+\underline{r}_{jk}-i} \; * \cdots *}_{p_k}\Bigg] ,$$

where, for notational purpose again, we denoted $n_1+\ldots+n_j$ by $\underline{j}$ and $p_1 + \ldots + p_{k-1}$ by $\underline{p}_{k-1}$.
It is easily verified that up to a minus sign this row coincides with row number $n_1+\ldots+n_j$ of F_4, which demonstrates the final part of the proof. □

5. CONCLUSIONS

In this paper we solved the deadbeat-output state-feedback control problem with internal stability. The obtained results extend straightforwardly the existing literature on this subject, where restrictions were made on the considered system. Here we dropped all these assumptions and pursued a geometric approach to solve the problem. To obtain an expression for the minimal settling time and a minimal time controller the analysis performed by Kimura et al. (1981) turned out to be very useful. Using a similar argument we derived an algorithm for the construction of a minimal time controller. By considering a particular case their result is obtained.
The big advantage of our geometric approach is that we can use these results to relate least squares minimization problems to the construction of minimal-time deadbeat controllers.
Starting from the transformed state-space description of the system we showed, using Luenberger's phase canonical form, that we can always construct a minimal-time deadbeat controller which can be reinterpreted in a least squares setting. Furthermore we showed that this type of controller

has also some transient minimal-time output-deadbeat properties. Its minimal settling time turned out to be directly computable from the transformed system.

APPENDIX

Proof of lemma 10

It is easiliy verified that for m = 0 and m = 1 then claim holds whatever n is.

So without loss of generality we may assume $\underline{r} \geq 2$.

Now consider A^k for some $2 \leq k \leq \underline{r}$. Obviously this matrix has the structure $\begin{bmatrix} J_1^k & D_k \\ 0 & J_2^k \end{bmatrix}$.

Since $A^k = A.A^{k-1}$, it is not difficult to see that the rows of matrix D_k are recursively obtained as follows:

$$i^{th}\text{-row } D_k = (i+1)^{th} - \text{row } D_{k-1} + i^{th}\text{-row } D_{\rightarrow k-1},\ 1 \leq i \leq \underline{r}. \qquad \text{(i)}$$

Here $D_{\rightarrow k} \in \mathbb{R}^{n \times m}$ is the matrix which first k columns are zero and the other columns consist of the m-k first columns of D. Using this relationship (i) we have (as can be shown by induction) that D_k equals

$$\begin{bmatrix}
* & \cdot & \cdot & \cdot & & * \\
\vdots & & & & & \vdots \\
* & \cdot & \cdot & \cdot & & * \\
f_1 & f_2+d_{n-11} & f_3+d_{n-12}+d_{n-21} & \cdots & & f_m + \sum_{i=1}^{k-1} d_{n-im-i} \\
0 & \ddots & \ddots & \ddots & & \\
\vdots & & & & & \vdots \\
 & & & f_3+d_{n-12}+d_{n-21} & & f_{m-k-3}+d_{n-1m-k+2}+d_{n-2m-k+1} \\
 & & & f_2+d_{n-11} & & f_{m-k+2}+d_{n-1m-k+1} \\
0 & \cdot & \cdot\ \cdot\ 0 & f_1 & f_2 & f_{m-k+1}
\end{bmatrix} \qquad \text{(ii)}$$

Here a '*' denotes an arbitrary real number.

To derive the final result we distinguish two cases: $n \geq m$ and $n < m$, respectively.

If $n \geq m$, we have from the above expression (ii) that

$$A^{\underline{r}} = \left[\begin{array}{c|c} J_1^{\underline{r}} & D_{\underline{r}} \\ \hline 0_{m\times n} & 0_{m\times m} \end{array}\right],$$

with

$$D_{\underline{r}} := \begin{bmatrix} * & \cdot & \cdot & * \\ \vdots & & & \vdots \\ * & \cdot & \cdot & * \\ f_1 & \cdots & & f_m + \sum_{i=1}^{m-1} d_{n-im-i} \\ 0 & \ddots & & \\ \vdots & & \ddots & \vdots \\ 0 & \cdot\ \cdot\ \cdot & 0 & f_1 \end{bmatrix}$$

Again using the relationship $A^{\underline{r}+k} = A.A^{\underline{r}+k-1}$ it is now inductively easily verified that for $1 \le k \le n-m$

$$A^{\underline{r}+k} = \left[\begin{array}{c|c} J_1^{\underline{r}+k} & D_{\underline{r}+k} \\ \hline 0_{m\times n} & 0_{m\times m} \end{array}\right],$$

with

$$D_{\underline{r}+k} := \begin{array}{l} \left.\begin{array}{l} \\ \\ \end{array}\right\} n-m-k \\ \\ \\ \\ \\ \\ \left.\begin{array}{l} \\ \\ \end{array}\right\} k \end{array} \begin{bmatrix} * & \cdot & \cdot & \cdot & \cdot & * \\ \vdots & & & & & \vdots \\ * & \cdot & \cdot & \cdot & \cdot & * \\ f_1 & \cdots & & & & f_m + \sum_{i=1}^{m-1} d_{n-im-i} \\ 0 & \ddots & & & & \\ \vdots & & & & \ddots & \vdots \\ 0 & \cdot & \cdot & 0 & & f_1 \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}$$

By substituting k = n-m, we then get the in this lemma advertised result. The case $n < m$ is proved similarly. First we note that

$$A^{\underline{r}} = \left[\begin{array}{c|c} 0_{n\times n} & D_{\underline{r}} \\ \hline 0_{m\times m} & J_2^m \end{array}\right]$$

with

$$D_{\underline{r}} = \begin{bmatrix} f_1 & f_2+d_{n-11} & & \cdots & f_m + \sum_{i=1}^{n-1} d_{n-im-i} \\ 0 & \ddots & \ddots & & \\ \vdots & & f_2+d_{n-11} & & \vdots \\ 0 & \cdots\ 0 & f_1 & f_2 \ \cdot\ \cdot & f_{m-n+1} \end{bmatrix}$$

Applying now the relationship $A^{\underline{r}+k} = A.A^{\underline{r}+k-1}$ yields for $1 \le k \le m-n$:

$$A^{\underline{r}+k} = \left[\begin{array}{c|c} 0_{n\times n} & D_{\underline{r}+k} \\ \hline 0_{m\times n} & J_2^{\underline{r}+k} \end{array}\right]$$

with

$$D_{\underline{r}+k} = \left[\begin{array}{ccccccc} 0 & \cdots & 0 & f_1 & f_2+d_{n-11} & \cdots & f_{m-k} + \sum_{i=1}^{n-1} d_{n-im-k-i} \\ \vdots & & \vdots & & f_2+d_{n-11} & & \vdots \\ \underbrace{0 \quad \cdots \quad 0}_{k} & & & & f_1 \quad f_2 & \cdot\ \cdot & f_{m-k-n+1} \end{array}\right]$$

Taking k = m-n yields then the stated result. □

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